LAB-2 Preliminary

Please follow the instructions in the document and mail your pdf-files to the TA of your section

artunsel@gmail.com,

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Please name your pdf files as in the given example file:

Mehmet-Ali-Demir-111211102-lab-1-preliminary-G-3.pdf

Mehmet-Ali-Demir-111211102-lab-1-labreport-G-3.pdf

ALSO STATE YOUR SECTION in the E-MAIL, [there are 3 sections]

section-1 TA: Mehmet Karahan,

section-2 TA: Mehmet Karahan,

section-3 TA: Artun Sel.

PLEASE READ "Important Rules" section at the end of this document before submitting your document.

THE DEADLINE: Friday, October 28, 2022, 20:00.

WARNING: Any work submitted at any time within the first 24 hours following the published submission deadline will receive a penalty of 10% of the maximum amount of marks available. Any work submitted at any time between 24 hours and up to 48 hours late will receive a deduction of 20% of the marks available.

Contents

TASK 1

Find the transfer function of the plant whose dynamics are given by

$$\boxed{\ddot{x}(t) + 3\dot{x}(t) + 2x(t) = u(t)}, \boxed{\begin{matrix} initial\ conditions \\ x(0) = 0 \\ \dot{x}(0) = 0 \end{matrix}}, \boxed{\begin{matrix} input = u(t) \\ output = x(t) \end{matrix}}, \boxed{\frac{X(s)}{U(s)}} = ?$$

Example for Task 1:

Find the transfer function of the system, whose dynamics are given by,

$$\boxed{\dddot{x}(t) + 5\ddot{x}(t) - 7\dot{x}(t) + 3x(t) = u(t)}, \boxed{\begin{matrix} initial\ conditions \\ x(0) = 0 \\ \dot{x}(0) = 0 \\ \ddot{x}(0) = 0 \end{matrix}}, \boxed{\begin{matrix} input = u(t) \\ output = x(t) \end{matrix}}, \boxed{\frac{X(s)}{U(s)}} = ?$$

Solution:

$$\dddot{x}(t) + 5\ddot{x}(t) - 7\dot{x}(t) + 3x(t) = u(t)$$

By taking the Laplace transform of this Differential Equation, we get

$$\mathcal{L}\{\dddot{x}(t)\} = s^3X(s)$$

$$\mathcal{L}\{5\ddot{x}(t)\} = 5s^2X(s)$$

$$\mathcal{L}\{-7\dot{x}(t)\} = -7sX(s)$$

$$\mathcal{L}\{3x(t)\} = 3X(s)$$

$$\mathcal{L}\{u(t)\} = U(s)$$

$$\mathcal{L}\{\dddot{x}(t) + 5\ddot{x}(t) - 7\dot{x}(t) + 3x(t)\} = \mathcal{L}\{u(t)\}$$

$$\mathcal{L}\{\dddot{x}(t)\} + \mathcal{L}\{5\ddot{x}(t)\} + \mathcal{L}\{-7\dot{x}(t)\} + \mathcal{L}\{3x(t)\} = \mathcal{L}\{u(t)\}$$

$$s^3X(s) + 5s^2X(s) + -7sX(s) + 3X(s) = U(s)$$

$$X(s)[s^3 + 5s^2 - 7s + 3] = U(s)$$

And finally, the transfer function is determined as

$$\frac{X(s)}{U(s)} = \frac{1}{[s^3 + 5s^2 - 7s + 3]}.$$

TASK 2

Find the poles and zeros of the transfer function of the plant in TASK-1.

Example-1 for Task 2: There are 5 transfer functions, and their corresponding poles are given below.

Transfer Functions	Denominator of the Transfer Function	Characteristic Equation of the system	Poles of the system
$T_1(s) = \frac{1}{s+1}$	$s + 1$	$s + 1 = 0$	$\{-1\}$
$T_2(s) = \frac{s+2}{s^2+3s+2}$	$s^2 + 3s + 2$	$s^2 + 3s + 2 = 0$ $(s+1)(s+2) = 0$	$\{-1, -2\}$
$T_3(s) = \frac{1}{(s+1)^2}$	$(s+1)^2$	$(s+1)^2 = 0$	$\{-1, -1\}$
$T_4(s) = \frac{1}{(s+1)(s+2)(s+3)}$	$(s+1)(s+2)(s+3)$	$(s+1)(s+2)(s+3) = 0$	$\{-1, -2, -3\}$
$T_5(s) = \frac{1}{(s-1)(s+2)}$	$(s-1)(s+2)$	$(s-1)(s+2) = 0$	$\{1, -2\}$

Example-2 for Task 2: There are 5 transfer functions, and their corresponding zeros are given below.

Transfer Functions	Numerator of the Transfer Function	Zeros of the system
$T_1(s) = \frac{1}{s+1}$	1	$\{\}$ It is an empty set
$T_2(s) = \frac{s+2}{s^2+3s+2}$	$s + 2$	$\{-2\}$
$T_3(s) = \frac{(s+2)^2}{(s+1)^3}$	$(s+2)^2$	$\{-2, -2\}$
$T_4(s) = \frac{1}{(s+1)(s+2)(s+3)}$	1	$\{\}$ It is an empty set
$T_5(s) = \frac{(s+1)(s-2)}{(s-1)(s+2)(s+3)}$	$(s+1)(s-2)$	$\{-1, 2\}$

TASK 3

Use MATLAB's "impulse" command to obtain the impulse-response of the system given in TASK-1. Plot the impulse response.

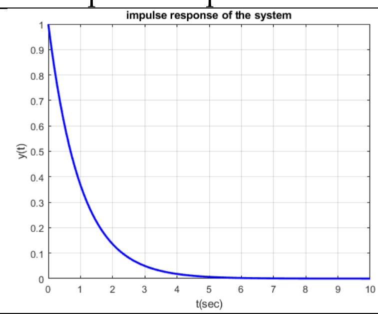
Example for Task 3: There are 3 systems and their corresponding impulse-responses listed below.

$$T_1(s) = \frac{1}{s+1}$$

Matlab Code

```
T_1=tf([1],[1,1]);          % define the TransferFunction "1/(s+1)"
[y,t] = impulse(T_1,[10]);  % get the impulse response for "10 seconds"
fig1=figure(1);
plot(t,y,'LineWidth',[2],'Color','b');
xlabel('t(sec)');
ylabel('y(t)');
fig1.Color=[1,1,1]; % to make the background "white"
title('impulse response of the system');
grid on;
```

Impulse-response Plot

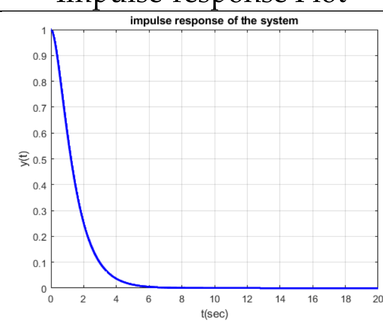


$$T_2(s) = \frac{s+3}{s^2+3s+2}$$

Matlab Code

```
T_2=tf([1,3],[1,3,2]);          % define the TransferFunction "(s+3)/(s^2+3s+2)"
[y,t] = impulse(T_2,[20]);  % get the impulse response for "20 seconds"
fig1=figure(1);
plot(t,y,'LineWidth',[2],'Color','b');
xlabel('t(sec)');
ylabel('y(t)');
fig1.Color=[1,1,1]; % to make the background "white"
title('impulse response of the system');
grid on;
```

Impulse-response Plot

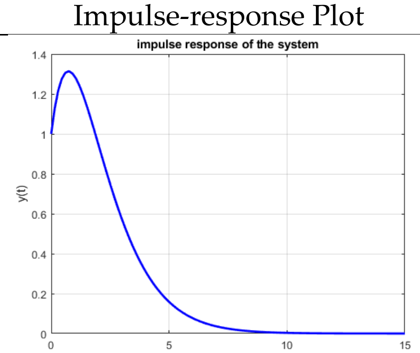


$$T_3(s) = \frac{(s+2)^2}{(s+1)^3}$$

Matlab Code

```
numerator3=conv([1,2],[1,2]);% to calculate the coefficients of (s+2)^2
denaminator3=conv([1,1],conv([1,1],[1,1])); % to calculate the coefficients of (s+1)^3
T_3=tf(numerator3,denaminator3);% define the TransferFunction "(s+2)^2/(s+1)^3"
[y,t] = impulse(T_3,[15]);  % get the impulse response for "15 seconds"
fig1=figure(1);
plot(t,y,'LineWidth',[2],'Color','b');
xlabel('t(sec)');
ylabel('y(t)');
fig1.Color=[1,1,1]; % to make the background "white"
title('impulse response of the system');
grid on;
```

Impulse-response Plot



TASK 4

Use MATLAB's "step" command to obtain the step-response of the system given in TASK-1. Plot the step response.

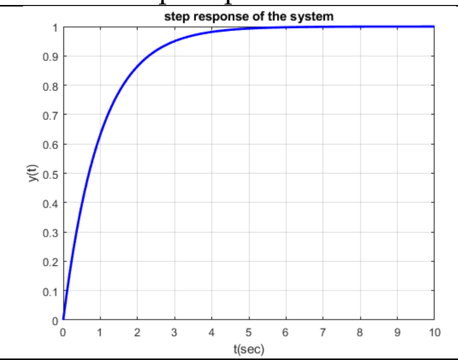
Example for Task 4: There are 3 systems and their corresponding step-responses listed below.

$$T_1(s) = \frac{1}{s+1}$$

Matlab Code

```
T_1=tf([1],[1,1]);          % define the TransferFunction "1/(s+1)"
[y,t] = step(T_1,[10]);  % get the step response for "10 seconds"
fig1=figure(1);
plot(t,y,'LineWidth',[2],'Color','b');
xlabel('t(sec)');
ylabel('y(t)');
fig1.Color=[1,1,1]; % to make the background "white"
title('step response of the system');
grid on;
```

Step-response Plot

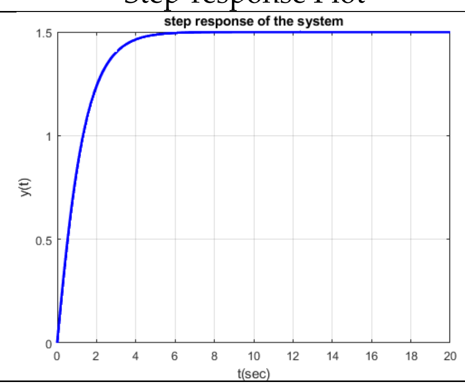


$$T_2(s) = \frac{s+3}{s^2+3s+2}$$

Matlab Code

```
T_2=tf([1,3],[1,3,2]);          % define the TransferFunction
"(s+3)/(s^2+3s+2)"
[y,t] = step(T_2,[20]);  % get the step response for "20 seconds"
fig1=figure(1);
plot(t,y,'LineWidth',[2],'Color','b');
xlabel('t(sec)');
ylabel('y(t)');
fig1.Color=[1,1,1]; % to make the background "white"
title('step response of the system');
grid on;
```

Step-response Plot

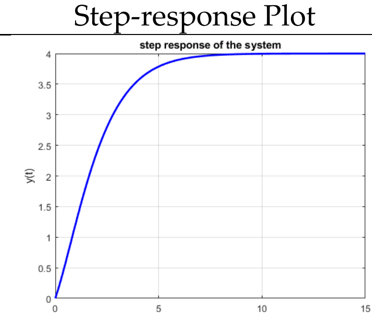


$$T_3(s) = \frac{(s+2)^2}{(s+1)^3}$$

Matlab Code

```
numerator3=conv([1,2],[1,2]);           % to calculate the coefficients of (s+2)^2
denaminator3=conv([1,1],conv([1,1],[1,1])); % to calculate the coefficients of (s+1)^3
T_3=tf(numerator3,denaminator3);% define the TransferFunction "(s+2)^2/(s+1)^3"
[y,t] = step(T_3,[15]);  % get the step response for "15 seconds"
fig1=figure(1);
plot(t,y,'LineWidth',[2],'Color','b');
xlabel('t(sec)');
ylabel('y(t)');
fig1.Color=[1,1,1]; % to make the background "white"
title('step response of the system');
grid on;
```

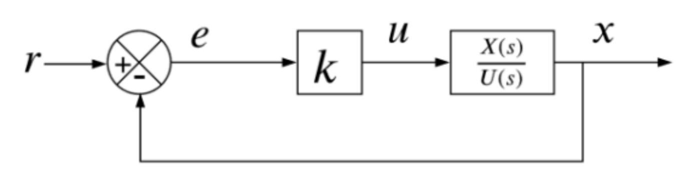
Step-response Plot

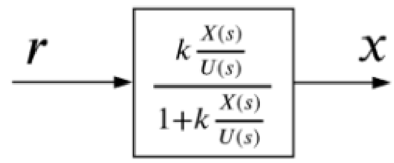


TASK 5

Find the closed-loop TF for a given block diagram. [Find the transfer function $\frac{X(s)}{R(s)}$]

In the block diagram, $\boxed{k}$ represents a constant-gain.



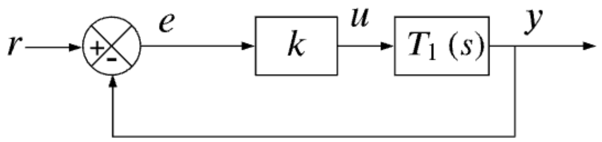


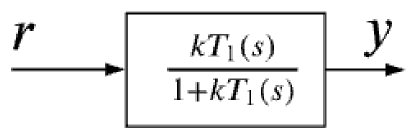
$$\frac{k\frac{X(s)}{U(s)}}{1+k\frac{X(s)}{U(s)}}$$

Example for Task 5: There are 3 closed-loop systems and their corresponding closed-loop transfer functions listed below.

Example-1 for Task 5

Find the closed-loop transfer function for the given block diagram below.
[Find the transfer function $\frac{Y(s)}{R(s)}$]
Where the plant transfer function is given by $\boxed{T_1(s) = \frac{1}{s+1}}$.





$$\frac{kT_1(s)}{1+kT_1(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{kT_1(s)}{1 + kT_1(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{k\frac{1}{s+1}}{1 + k\frac{1}{s+1}}$$

$$\frac{Y(s)}{R(s)} = \frac{k}{(s+1) + k}$$

And finally, the closed-loop transfer function is obtained as

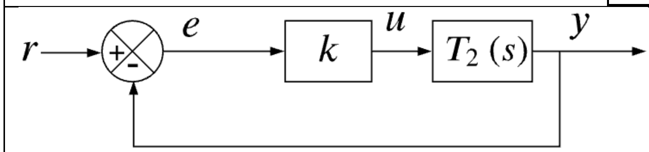
$$\frac{Y(s)}{R(s)} = \frac{k}{s + (k+1)}.$$

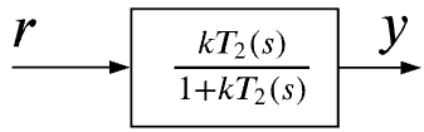
Example-2 for Task 5

Find the closed-loop transfer function for the given block diagram below.

[Find the transfer function $\frac{Y(s)}{R(s)}$]

Where the plant transfer function is given by $\boxed{T_2(s) = \frac{s+3}{s^2+3s+2}}$.





$$\frac{Y(s)}{R(s)} = \frac{kT_2(s)}{1 + kT_2(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{k\frac{s+3}{s^2+3s+2}}{1 + k\frac{s+3}{s^2+3s+2}}$$

$$\frac{Y(s)}{R(s)} = \frac{k(s+3)}{(s^2+3s+2) + k(s+3)}$$

And finally, the closed-loop transfer function is obtained as

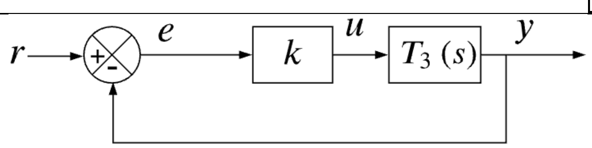
$$\frac{Y(s)}{R(s)} = \frac{ks + 3k}{s^2 + (3+k)s + (2+3k)}.$$

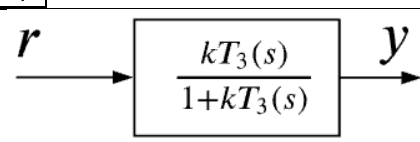
Example-3 for Task 5

Find the closed-loop transfer function for the given block diagram below.

[Find the transfer function $\frac{Y(s)}{R(s)}$]

Where the plant transfer function is given by $\boxed{T_3(s) = \frac{(s+2)^2}{(s+1)^3}}$.





$$\frac{Y(s)}{R(s)} = \frac{kT_3(s)}{1 + kT_3(s)}$$

$$\frac{Y(s)}{R(s)} = \frac{k\frac{(s+2)^2}{(s+1)^3}}{1 + k\frac{(s+2)^2}{(s+1)^3}}$$

$$\frac{Y(s)}{R(s)} = \frac{k(s+2)^2}{(s+1)^3 + k(s+2)^2} \rightarrow \frac{k(s^2+4s+4)}{[s^3+3s^2+3s+1] + k(s^2+4s+4)}$$

And finally, the closed-loop transfer function is obtained as

$$\frac{Y(s)}{R(s)} = \frac{[k]s^2 + [4k]s + [4k]}{[1]s^3 + [3+k]s^2 + [3+4k]s + [1+4k]}.$$

Important Rules

The following is the list of the rules that must be followed. The failure of following the rules listed below will be resulted in point-deduction as stated in the table.

No.	Rule	Corresponding point-deduction for the failure of following the rule
01	The document must be mailed to the TA of the section	5 pt.
02	The pdf file must be named as stated at the top of the document	5 pt.
03	The file must be in pdf format	5 pt.
04	Section-name must be stated in the mail that is to be sent to submit the **lab-report** or **preliminary** document	5 pt.
05	The deadline must be met.	10 pt. for each day after the deadline
06	The file must be prepared in digital form. MSword or Latex must be used.	5 pt.
07	All plots must be on a white background and the lines must be clearly visible. The names of the signals in the plot must be stated [either by using legend or by using appropriate Figure Naming such as "Figure 1: (red) input signal, (blue) output signal"]	3 pt.
08	All figures must be numbered.	3 pt.
09	All tables must be numbered.	3 pt.
10	All equations must be numbered.	3 pt.
11	References must be added. Only books are allowed. Do not use internet sources. Example references: [1] "Modern Control Engineering 5th Ed", Ogata K., 2010, Prentice Hall [2] "Linear Systems Theory 2nd Ed", Hespanha J., 2018, Princeton Press	3 pt.
12	Font style must be consistent. Times-New-Roman or Palatino-Linotype must be used. Font size must be 11.	3 pt.
13	Interpret the findings in each task accordingly.	5 pt.