LAB-4 EXPERIMENT

Please follow the instructions in the document and mail your pdf-files to the TA of your section

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Please name your pdf files as in the given example file:

Mehmet-Ali-Demir-111211102-lab-1-preliminary-G-3.pdf

Mehmet-Ali-Demir-111211102-lab-1-labreport-G-3.pdf

ALSO STATE YOUR SECTION in the E-MAIL, [there are 3 sections]

section-1 TA: Mehmet Karahan,

section-2 TA: Mehmet Karahan,

section-3 TA: Artun Sel.

PLEASE READ "Important Rules" section at the end of this document before submitting your document.

THE DEADLINE: Friday, November 18, 2022, 20:00.

WARNING: Any work submitted at any time within the first 24 hours following the published submission deadline will receive a penalty of 10% of the maximum amount of marks available. Any work submitted at any time between 24 hours and up to 48 hours late will receive a deduction of 20% of the marks available

Contents

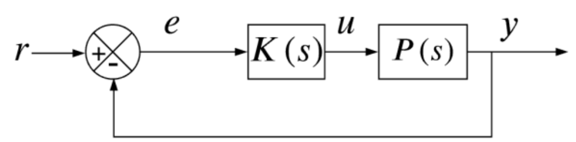
Problem 1

This is a demo-problem. Try to understand this problem and then solve the other problems after that.

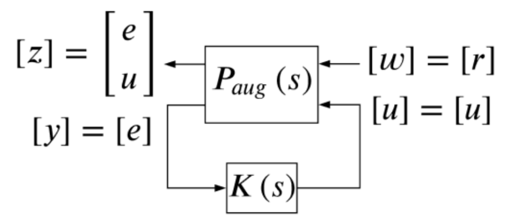
Task-1

For a given Plant transfer-function $P(s)$,

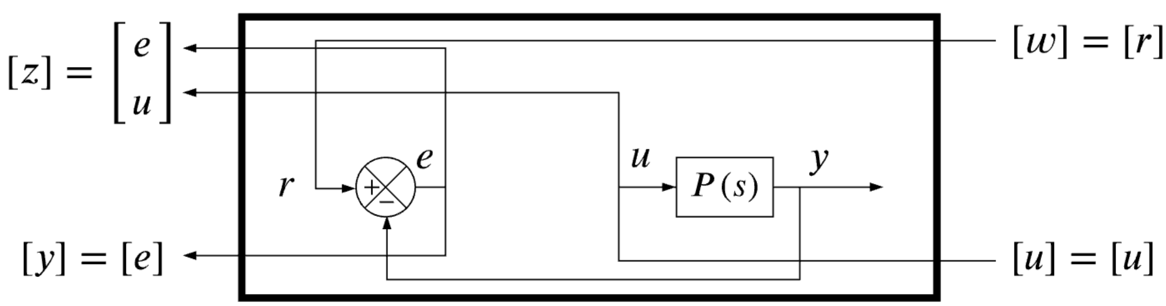
$$P(s) = \frac{(s - p_3)}{s^2 + p_1 s - p_2}, p_1 = 1, p_2 = 1, p_3 = 1$$



Closed-loop control system block diagram



General block diagram



$P_{aug}(s)$ system block diagram

Find $P_{aug}(s)$ transfer-function.

SOLUTION:

To find $P_{aug}(s)$ system, the following matlab script is used.

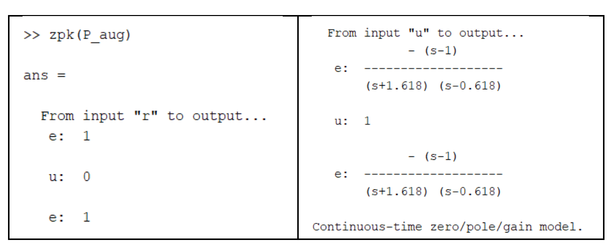
Matlab code

```
clear all,close all,clc;
%% define the parameters
p1=1;    p2=1;    p3=1;
%% define the PLANT
Plant=tf([1,-p3],[1,p1,-p2]);
%% constructing the P-AUG(augmented)
Plant.u='u';
Plant.y='y';

Summer_1=sumblk('e=r-y');

P_aug =
connect(Plant,Summer_1,{'r','u'},{'e','u','e'})
zpk(P_aug)
```

Code output



```
>> zpk(P_aug)

ans =

  From input "r" to output...
   e:  1

   u:  0

   e:  1

  From input "u" to output...
          - (s-1)
   e:  -------------------
       (s+1.618) (s-0.618)

   u:  1

          - (s-1)
   e:  -------------------
       (s+1.618) (s-0.618)

Continuous-time zero/pole/gain model.
```

From this one can obtain that the $P_{aug}(s)$ system is

$$P_{aug}(s) = \begin{bmatrix} 1 & \frac{-(s-1)}{(s+1.618)(s-0.618)} \\ 0 & 1 \\ 1 & \frac{-(s-1)}{(s+1.618)(s-0.618)} \end{bmatrix}$$

Notice the relation between the $\begin{bmatrix} \boxed{w = r} \\ \boxed{u = u} \end{bmatrix}$ and $\begin{bmatrix} \boxed{z = \begin{bmatrix} e \\ u \end{bmatrix}} \\ \boxed{y = [e]} \end{bmatrix}$

And

$$\begin{bmatrix} \boxed{w = r} \\ \boxed{u = u} \end{bmatrix} = \begin{bmatrix} 1 & \frac{-(s-1)}{(s+1.618)(s-0.618)} \\ 0 & 1 \\ 1 & \frac{-(s-1)}{(s+1.618)(s-0.618)} \end{bmatrix} \begin{bmatrix} \boxed{z = \begin{bmatrix} e \\ u \end{bmatrix}} \\ \boxed{y = [e]} \end{bmatrix}$$

Where,

Signal	Signal name
w	Exogenous input
u	Control input
z	Regulated output
y	Measured output

The description of the signals

Signal	Signal name	Description
w	Exogenous input	The signals the disturb the operation. For example, reference signal can be considered as a disturbance because it is an external signal that is not generated by the CONTROLLER.
u	Control input	The signal that is generated by the CONTROLLER.
z	Regulated output	The output of the augmented system. These are the signals that are required to be minimized. For example, error signal is required to be minimized. In addition to that input signal to the PLANT is also desired to be minimized because if we can achieve the same objective with less input-energy we should choose the less-input-energy consuming design. Let us say there are 2 control designs for a motor speed control application. The 1st controller requires +30V (as highest control signal) and the 2nd controller required +15V (as highest control signal), in this case the 2nd controller is better.
y	Measured output	These are the signals that CONTROLLER uses to generate the control-signal. It can also be considered as the input signals to the CONTROLLER.

Task-2

Design an hinf controller. (hinf = h-infinity = $\mathcal{H}_\infty$ controller)

SOLUTION

The necessary matlab code to obtain the $\mathcal{H}_\infty$ controller is given as,

Matlab code	Code output

<pre>clear all,close all,clc;
%% define the parameters
p1=1;
p2=1;
p3=1;
%% define the PLANT
Plant=tf([1,-p3],[1,p1,-p2]);
%% constructing the P-AUG(augmented)
Plant.u='u';
Plant.y='y';
 	
Summer_1=sumblk('e=r-y');
 	
P_aug =
connect(Plant,Summer_1,{'r','u'},{'e','u','e'});
zpk(P_aug)
%%
% P_aug=[1,-Plant;0,1;1,-Plant];
% [K,CL,gamma] = hinfsyn(P,nmeas,ncont)
[K,CL,gamma] = hinfsyn(P_aug,1,1);
tf(K)
zpk(K)</pre>
	<pre>>> tf(K)
	
	ans =
	
	    -787.9 s - 1275
	  -------------------- 	
	  s^2 + 91.77 s + 1088
	
	Continuous-time transfer function.
	
	>> zpk(K)
	
	ans =
	
	   -787.86 (s+1.618)
	  ------------------- 	
	  (s+13.98) (s+77.79)
	
	Continuous-time zero/pole/gain model.</pre>

As a result of this code-output, the CONTROLLER is obtained as,

$$K(s) = \frac{-787.86(s + 1.618)}{(s + 13.98)(s + 77.79)}$$

Task-3

Find the $T_{zw}(s)$ transfer-function for the $K(s)$ (controller transfer function) that you obtained in the previous task.

$T_{zw}(s)$ is the transfer-function of the system between disturbance-inputs (w) to regulated-outputs (z). $T_{zw}(s)$ expression is dependent on $K(s)$ and $P_{aug}(s)$ transfer-functions.

SOLUTION

To find $T_{zw}(s)$ we can directly use MATLAB. The following matlab code is used to find $T_{zw}(s)$.

Matlab code	Code output
See code below	See output below

Matlab code

```
clear all,close all,clc;
%% define the parameters
p1=1;
p2=1;
p3=1;
%% define the PLANT
Plant=tf([1,-p3],[1,p1,-p2]);
%% Find P-AUG(augmented)
Plant.u='u';
Plant.y='y';

Summer_1=sumblk('e=r-y');

P_aug =
connect(Plant,Summer_1,{'r','u'},{'e','u','e'});
zpk(P_aug)

P_aug.u={'w','u'}
P_aug.y={'z1','z2','y'}
%% Find K(s)
% P_aug=[1,-Plant;0,1;1,-Plant];
% [K,CL,gamma] = hinfsyn(P,nmeas,ncont)
[K,CL,gamma] = hinfsyn(P_aug,1,1);
K.u='y';
K.y='u';
% tf(K)
% zpk(K)
zpk(minreal(CL))
% sigma(CL)
%% Find Tzw(s)
Tzw=connect(P_aug,K,{'w'},{'z1','z2'})
zpk(Tzw)
zpk(minreal(Tzw))
```

Code output

```
ans =

  From input "w" to output...
         (s+1.618) (s-0.618) (s+13.98) (s+77.79)
   z1:  ---------------------------------------
         (s+88.42) (s+2.116) (s+1.618) (s+0.618)

              -787.86 (s-0.618) (s+1.618)^2
   z2:  ---------------------------------------
         (s+88.42) (s+2.116) (s+1.618) (s+0.618)

Continuous-time zero/pole/gain model.

1 state removed.

ans =

  From input "w" to output...
         (s+77.79) (s+13.98) (s-0.618)
   z1:  -----------------------------
         (s+88.42) (s+2.116) (s+0.618)

          -787.86 (s-0.618) (s+1.618)
   z2:  -----------------------------
         (s+88.42) (s+2.116) (s+0.618)

Continuous-time zero/pole/gain model.
```

As a result of this code-output, $T_{zw}(s)$ is obtained as,

$$T_{zw}(s) = \begin{bmatrix} \boxed{\dfrac{(s+77.79)(s+13.98)(s-0.618)}{(s+88.42)(s+2.116)(s+0.618)}} \\ \boxed{\dfrac{-787.86(s-0.618)(s+1.618)}{(s+88.42)(s+2.116)(s+0.618)}} \end{bmatrix}$$

Since there is 1 disturbance-input and 2 regulated-outputs, $T_{zw}(s)$ is a 2x1 transfer-function.

Task-4

Find the gamma. GAMMA is the GAIN from disturbance-signals (w) to the regulated-outputs (z). We want this GAIN to be minimized. If we can minimize this GAIN, we can say that we have a controller that is robust to the disturbance signals.

$$\gamma = \min_{K(s)} \|T_{zw}(s)\|_{\mathcal{H}_\infty}$$

Where, $T_{zw}(s)$ is the transfer function from disturbance-signals (w) to regulated-output (z). γ value is the "norm" of this transfer-function. If we can minimize γ, that means we minimize the norm of the transfer-function between disturbance signals to regulated-outputs. In other words, if we can minimize γ, we can minimize the effect of disturbance-inputs to the regulated-outputs.

SOLUTION

To find GAMMA we can directly use MATLAB. The following matlab code is used to find GAMMA.

Matlab code

```
clear all,close all,clc;
%% define the parameters
p1=1;
p2=1;
p3=1;
%% define the PLANT
Plant=tf([1,-p3],[1,p1,-p2]);
%% Find P-AUG(augmented)
Plant.u='u';
Plant.y='y';

Summer_1=sumblk('e=r-y');

P_aug =
connect(Plant,Summer_1,{'r','u'},{'e','u','e'})
;
zpk(P_aug)

P_aug.u={'w','u'}
P_aug.y={'z1','z2','y'}
%% Find K(s)
% P_aug=[1,-Plant;0,1;1,-Plant];
% [K,CL,gamma] = hinfsyn(P,nmeas,ncont)
[K,CL,gamma] = hinfsyn(P_aug,1,1);
K.u='y';
K.y='u';
% tf(K)
% zpk(K)
zpk(minreal(CL))
% sigma(CL)
%% Find Tzw(s)
Tzw=connect(P_aug,K,{'w'},{'z1','z2'})
zpk(Tzw)
zpk(minreal(Tzw))
%% Find GAMMA
% [ninf,fpeak] = hinfnorm(sys,tol)
[GAMMA,fpeak] = hinfnorm(Tzw,1e-3)
```

Code output

```
>> [GAMMA,fpeak] = hinfnorm(Tzw,1e-3)

GAMMA =

    8.9550


fpeak =

     0
```

As a result of the code-output, the GAMMA value is obtained as

$$\gamma = 8.9950$$

Task-5

Test this controller for the parameter uncertainty case where the parameters take values between given lower and upper limits. Find the corresponding GAMMA value for each parameter combination and determine the worst (greatest) GAMMA value.

$p_{1_{nom}} = 1$	$p_{2_{nom}} = 1$	$p_{3_{nom}} = 1$
$p_1 \in [p_{1_{nom}} - 0.01, p_{1_{nom}} + 0.01]$	$p_2 \in [p_{2_{nom}} - 0.01, p_{2_{nom}} + 0.01]$	$p_3 \in [p_{3_{nom}} - 0.01, p_{3_{nom}} + 0.01]$

SOLUTION

For this problem, let us discretize the parameter range for each parameter and find GAMMA values for each parameter combinations. For this the given Algorithm is used.

Algorithm to solve task-5

$$p_{1_{vec}} = linspace(\boxed{p_{1_{nom}} - 0.01}, \boxed{p_{1_{nom}} + 0.01}, 10)$$

$$p_{2_{vec}} = linspace(\boxed{p_{2_{nom}} - 0.01}, \boxed{p_{2_{nom}} + 0.01}, 10)$$

$$p_{3_{vec}} = linspace\left(\boxed{p_{3_{nom}} - 0.01}, \boxed{p_{3_{nom}} + 0.01}, 10\right)$$

$$\gamma_{max} = -100$$

For $p_{1_{temp}} \in p_{1_{vec}}$

  For $p_{2_{temp}} \in p_{2_{vec}}$

    For $p_{3_{temp}} \in p_{3_{vec}}$

      compute $\boxed{P_{temp}(s) = \frac{(s - p_{3_{temp}})}{s^2 + p_{1_{temp}} s - p_{2_{temp}}}}$

      compute $\boxed{P_{aug_{temp}}(s)}$

      compute $\boxed{T_{zw_{temp}}(s)}$

      compute $\boxed{\gamma_{temp} = \left\| T_{zw_{temp}}(s) \right\|_{\mathcal{H}_\infty}}$

      compute $\boxed{\gamma_{max} = \max\{\gamma_{max}, \gamma_{temp}\}}$

    End

  End

End

Display $\boxed{\gamma_{max}}$

The necessary matlab code to solve this problem is the following,

Matlab code

```
clear all,close all,clc;
%% define the parameters
p1=1;
p2=1;
p3=1;
%% define the PLANT
Plant=tf([1,-p3],[1,p1,-p2]);
%% Find P-AUG(augmented)
Plant.u='u';
Plant.y='y';

Summer_1=sumblk('e=r-y');

P_aug = connect(Plant,Summer_1,{'r','u'},{'e','u','e'});
zpk(P_aug)

P_aug.u={'w','u'}
P_aug.y={'z1','z2','y'}
%% Find K(s)
% P_aug=[1,-Plant;0,1;1,-Plant];
% [K,CL,gamma] = hinfsyn(P,nmeas,ncont)
[K,CL,gamma] = hinfsyn(P_aug,1,1);
K.u='y';
K.y='u';
% tf(K)
% zpk(K)
zpk(minreal(CL))
% sigma(CL)
%% Find Tzw(s)
Tzw=connect(P_aug,K,{'w'},{'z1','z2'})
zpk(Tzw)
zpk(minreal(Tzw))
%% Find GAMMA
% [ninf,fpeak] = hinfnorm(sys,tol)
[GAMMA,fpeak] = hinfnorm(Tzw,1e-3)
%%

% p_range=[p_min,p_max]
p1_range=[p1-0.01,p1+0.01]
p2_range=[p2-0.01,p2+0.01]
p3_range=[p3-0.01,p3+0.01]
p1_vec=linspace(p1_range(1),p1_range(2),10);
p2_vec=linspace(p2_range(1),p2_range(2),10);
p3_vec=linspace(p3_range(1),p3_range(2),10);

max_Tzw_norm=0;
for i_1=1:1:length(p1_vec)
    for i_2=1:1:length(p2_vec)
        for i_3=1:1:length(p3_vec)
            p1_temp=p1_vec(i_1);
            p2_temp=p2_vec(i_2);
            p3_temp=p3_vec(i_3);
            % construct PLANT_temp
            Plant_temp=tf([1,-p3_temp],[1,p1_temp,-p2_temp]);
            % construct PAUG_temp
            Plant_temp.u='u';
            Plant_temp.y='y';
            Summer_1=sumblk('e=r-y');
            P_aug_temp = connect(Plant_temp,Summer_1,{'r','u'},{'e','u','e'});

            P_aug_temp.u={'w','u'};
            P_aug_temp.y={'z1','z2','y'};
%             P_aug_temp=[1,-Plant_temp;0,1;1,-Plant_temp];
            % construct Tzw_temp
            Tzw_temp=connect(P_aug_temp,K,{'w'},{'z1','z2'});
%             Tzw_temp=lft(P_aug_temp,K,1,1);
            % compute "norm of Tzw_temp"
            Tzw_norm_temp=hinfnorm(Tzw_temp);
            if Tzw_norm_temp>max_Tzw_norm
                disp([p1_temp,p2_temp,p3_temp,Tzw_norm_temp]);
                if max_Tzw_norm==Inf
                    return;
                end
            end
            max_Tzw_norm=max([max_Tzw_norm,Tzw_norm_temp]);
        end
    end
end
max_Tzw_norm
```

And the result is,

<table>
<tr>
<td>

```
    0.9900    0.9900    0.9900    9.2449

    0.9900    0.9922    0.9900    9.2595

    0.9900    0.9944    0.9900    9.2954

    0.9900    0.9967    0.9900    9.3825

    0.9900    0.9989    0.9900    9.5329

    0.9900    1.0011    0.9900    9.6875

    0.9900    1.0033    0.9900    9.8464

    0.9900    1.0056    0.9900   10.0099

    0.9900    1.0078    0.9900   10.1781

    0.9900    1.0100    0.9900   10.3514

    0.9922    1.0100    0.9900   10.3514


max_Tzw_norm =

   10.3514
```

</td>
<td>

And the code output is given in the right.
As a result of the matlab code-output,

$$\boxed{\gamma_{max} = 10.3514}$$

Is obtained.

CONCLUSION:
For the "nominal plant", we designed a controller $K(s)$, and this controller resulted in $\boxed{\gamma_{nominal} = 8.9950}$. however for the case where the parameter take values from their corresponding range meaning that have upper and lower bounds, The same controller $K(s)$ resulted in $\boxed{\gamma_{max} = 10.3514}$.
For each parameter combination we have calculated the corresponding GAMMA value and the worst-case is determined as $\boxed{\gamma_{max} = 10.3514}$

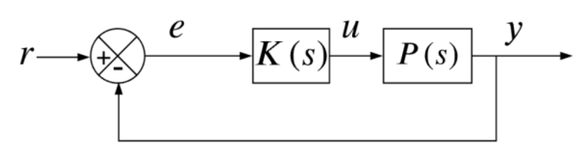
</td>
</tr>
</table>

Problem 2

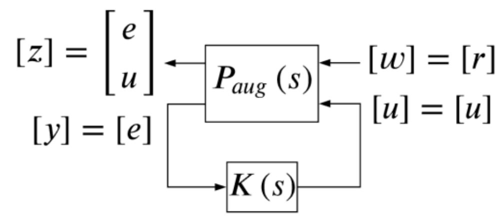
Task-1

For the given plant transfer-function $P(s)$,

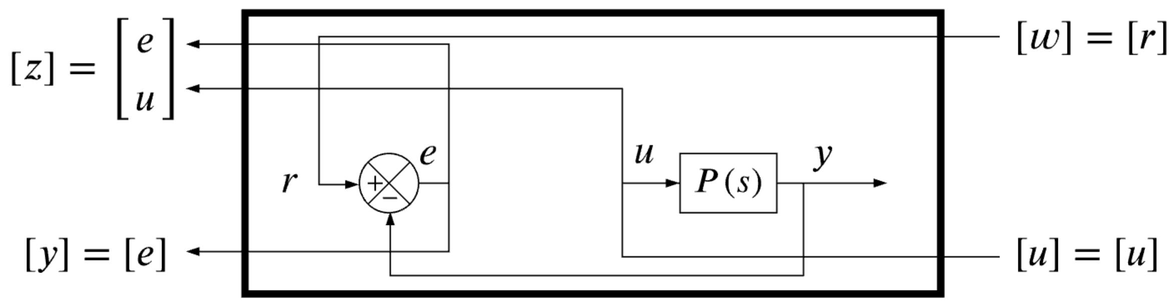
$$P(s) = \frac{(s + p_3)}{s^2 + p_1 s + p_2}, p_1 = 1, p_2 = 1, p_3 = 1$$



Closed-loop control system block diagram



General block diagram



$P_{aug}(s)$ system block diagram

Find $P_{aug}(s)$.

Task-2

Design an hinf controller. (hinf = h-infinity = $\mathcal{H}_\infty$ controller). Find $K(s)$ transfer-function.

Task-3

Find the $T_{zw}(s)$ transfer-function for the $K(s)$ (controller transfer function) that you obtained in the previous task.

$T_{zw}(s)$ is the transfer-function of the system between disturbance-inputs (w) to regulated-outputs (z). $T_{zw}(s)$ expression is dependent on $K(s)$ and $P_{aug}(s)$ transfer-functions.

Task-4

Find GAMMA γ. GAMMA value is the hinf-norm of the transfer-function $T_{zw}(s)$.

In other words,

$$\gamma = \|T_{zw}(s)\|_{\mathcal{H}_\infty}$$

Task-5

Test this controller for the parameter uncertainty case where the parameters take values between given lower and upper limits. Find the corresponding GAMMA value for each parameter combination and determine the worst (greatest) GAMMA value.

$p_{1_{nom}} = 1$	$p_{2_{nom}} = 1$	$p_{3_{nom}} = 1$
$p_1 \in [p_{1_{nom}} - 0.01, p_{1_{nom}} + 0.01]$	$p_2 \in [p_{2_{nom}} - 0.01, p_{2_{nom}} + 0.01]$	$p_3 \in [p_{3_{nom}} - 0.01, p_{3_{nom}} + 0.01]$

Important Rules

The following is the list of the rules that must be followed. The failure of following the rules listed below will be resulted in point-deduction as stated in the table.

No.	Rule	Corresponding point-deduction for the failure of following the rule
01	The document must be mailed to all 3 of the Teaching Assistants	5 pt.
02	The pdf file must be named as stated at the top of the document	5 pt.
03	The file must be in pdf format	5 pt.
04	Section-name must be stated in the mail that is to be sent to submit the **lab-report** or **preliminary** document	5 pt.
05	The deadline must be met.	10 pt. for each day after the deadline
06	The file must be prepared in digital form. MSword or Latex must be used.	5 pt.
07	All plots must be on a white background and the lines must be clearly visible. The names of the signals in the plot must be stated [either by using legend or by using appropriate Figure Naming such as "Figure 1: (red) input signal, (blue) output signal"]	3 pt.
08	All figures must be numbered.	3 pt.
09	All tables must be numbered.	3 pt.
10	All equations must be numbered.	3 pt.
11	References must be added. Only books are allowed. Do not use internet sources. Example references: [1] "Modern Control Engineering 5th Ed", Ogata K., 2010, Prentice Hall [2] "Linear Systems Theory 2nd Ed", Hespanha J., 2018, Princeton Press	3 pt.
12	Font style must be consistent. Times-New-Roman or Palatino-Linotype must be used. Font size must be 11.	3 pt.
13	Interpret the findings in each task accordingly.	5 pt.